THE AERODYNAMIC EQUILIBRIUM EQUATIONS
FOR THE LIFTING ROTOR

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Noah Carroll New

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PREFACE

MEANING OF SYMBOLS USED

a slope of section lift coefficient against section angle of attack

b number of blades

c blade-section chord

c_l section lift coefficient

r radius of blade element

t_n coefficient for the n^{th} term in the power series for the blade section chord

v_x tip-speed ratio $\frac{V_h \cos \alpha_R}{\Omega R}$

v_z inflow ratio $\frac{V_h \sin \alpha_R + \bar{V}_i}{\Omega R}$

w longitudinal increment of the induced velocity (along the x-axis)

x ratio of blade-element radius to rotor-blade radius (r/R)

y lateral increment of the induced velocity (along the y-axis)

C_{M1} blade root air moment coefficient for one blade

$C_{M_{X_1}}$ rolling moment coefficient for one blade

$C_{M_{Y_1}}$ pitching moment coefficient for one blade

C_{T_1} thrust coefficient for one blade

I_β moment of inertia of the blade about the flapping hinge

K_T right hand member of the thrust equation

K_R right hand member of the rolling moment equation

K_P right hand member of the pitching moment equation

K_M right hand member of the blade root moment equation

M_{CF}	moment due to centrifugal force
M_W	moment due to blade weight
M_1	blade root air moment of one blade
V	total resultant velocity of the blade element
V_i	induced velocity at a blade element
$\overline{V}_i$	value of the induced velocity at the rotor hub
V_h	true airspeed of the rotor along the flight path
$\overline{T}_1$	thrust for one blade
T_n	$\int_0^1 \frac{c}{R} x^{n-1} dx$ for $n = 1, 2, 3, \ldots n$
R	blade tip radius
α	blade-element angle of attack
α_R	rotor angle of attack
β_o	mean blade coning angle
θ	blade element pitch angle
θ_o	blade pitch angle at rotor hub
θ_{os}	$\sin \theta_o$
θ_{oc}	$\cos \theta_o$
θ_{11}	lateral component of cyclic pitch
θ_{12}	longitudinal component of cyclic pitch
θ_t	blade twist angle
θ_z	blade asimuth angle measured from downwind position in the direction of rotation
ρ	mass density of air
ϕ	inflow angle at blade element in plane perpendicular to blade-span axis
Ω	rotor angular velocity

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THE AERODYNAMIC EQUILIBRIUM EQUATIONS FOR THE LIFTING ROTOR

SUMMARY

This paper presents a theoretical development of the thrust, blade root air moment, pitching moment, and rolling moment equations for the lifting rotor. These equations are solved for the rotor coning angle, overall blade pitch angle, and cyclic pitch requirements necessary for rotor equilibrium.

The approximation used to simplify the equations so that integration is possible is that the blade element angles of attack can be assumed small angles. This in practice means that if appreciable portions of the rotor blades are not stalled the equations are valid. Also included is a method of representing the rotor blade planform so that the equilibrium equations can be applied to blades of arbitrary shape. A one point check of the theory with experimental data is given in the appendix.

INTRODUCTION

The equilibrium equations as presented here have been previously solved in detail by Castles.[1] His solution was based on the approximation that the blade inflow angle remain small. If instead of the inflow angle remaining small the blade element angle of attack is approximated by the sine of the angle, the equations, while more complicated, integrate with no difficulty. The blade element angles of attack will be small if they remain below the stalling angle of the blade section. The equations would thus appear to be applicable to the helicopter, convertaplane, and airplane propeller since their successful operation does not permit stalling of appreciable portions of the rotor blades.

It is believed that this paper presents for the first time a method for calculating the air force distribution on a rotor blade of arbitrary planform. This is accomplished by representing the rotor blade chord by an infinite power series. The coefficients of this series are never actually evaluated since the series when integrated represents the blade planform area, and it is in this form that it is used.

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[1]Castles, Walter, Jr., Introduction to the Aerodynamics of the Helicopter

DEVELOPMENT OF THE AERODYNAMIC EQUATIONS

THRUST EQUATION

Take the primary frame of reference as a right hand axis system in X, Y, and Z. Let the XY plane be parallel to the plane of zero first harmonic flapping. The positive direction of the Z-axis is in the direction of the rotor thrust. Let the XZ plane be so aligned that the velocity of the rotor along the flight path, V_h, lies in that plane. Define the angle this velocity vector makes with the X-axis as the rotor angle of attack, α_R, and let α_R be measured positive when the sine component of V_h is positive along the Z-axis. The positive X direction is in the same direction as that of the positive cosine component of V_h.

Letting V_i represent the induced inflow velocity of the rotor and resolving V_h into components along the X and Z axes gives the following components along the axes of the primary frame of reference:

X-axis	$V_h \cos \alpha_R$	(1)
Y-axis	0	(2)
Z-axis	$V_h \sin \alpha_R + V_i$	(3)

It is desirable to further resolve these velocity components into the components that act on the blade element, (1) radially along the blade-span axis, (2) along an axis perpendicular to both the blade-span axis and axis of zero flapping, and (3) an axis that is mutually perpendicular to the other two components. Let these three axes be respectively, (1) the blade-span axis, (2) the in-plane axis, and (3) the normal axis. To make the resolution of these components simpler a secondary frame of reference that will not replace but augment the primary frame of reference will be defined and used.

The secondary frame of reference is a system of spherical coordinates and has its origin common to that of the primary frame of reference. Let a blade element be at distance, r, from the origin measured along the blade-span axis. Let the blade-span axis at an azimuth angle, θ_z, and a coning angle, β_o. The azimuth angle will be measured in the positive direction about the Z-axis and have the X-axis for its reference line. The coning angle will be positive when the blade-span axis is below the XY plane, and as β_o is always small the usual approximations for small angles will be used. The component velocities along the X, Y, and Z axes when resolved into the normal axis in-plane axis are respectively

$$V \sin \phi = V_i + V_h \sin \alpha_R + V_h \beta_o \cos \alpha_R \cos \theta_z \qquad (4)$$

$$V \cos \phi = V_h \cos \alpha_R \sin \theta_z + \Omega r \qquad (5)$$

where $V =$ resultant velocity in the plane of the blade element at the blade element.

The angle θ in Figure 1, Appendix, is the blade-section pitch angle and is measured between the zero lift chord of the blade section and the XY plane.

The induced velocity, V_i, may be represented to a first approximation by an average value at the rotor hub upon which is superimposed a linear variation in both the X and Y directions. Such a representation may be written:

$$V_i = \overline{V}_i + \Omega r w \cos \theta_z + \Omega r y \sin \theta_z \qquad (6)$$

where $\overline{V}_i =$ average value over the rotor

$\Omega r w \cos \theta_z =$ increment at point r, θ_z due to linear variation along the X-axis

$\Omega r y \sin \theta_z =$ increment at point r, θ_z due to linear variation along the Y-axis

Define the following dimensionless coefficients

$$v_x = \frac{V_h \cos \alpha_R}{\Omega R} \qquad (7)$$

$$v_z = \frac{V_h \sin \alpha_R + \overline{V}_i}{\Omega R} \qquad (8)$$

$$x = \frac{r}{R} \qquad (9)$$

Rewriting the two velocity components given by equations (4) and (5) and using the above dimensionless coefficients together with the V_i of equation (6) gives the following velocity components along the normal and in-plane axes

$$V \sin \phi = \Omega R \left[v_z + v_x \beta_0 \cos \theta_z + x(w \cos \theta_z + y \sin \theta_z) \right] \tag{10}$$

$$V \cos \phi = \Omega R \left[v_x \sin \theta_z + x \right] \tag{11}$$

The lift equation may be written for a blade as follows:

$$L = \int_0^R c_\ell \left(\frac{\rho}{2} V^2 \right) c \, dr$$

where c = blade element chord

c_ℓ = section lift coefficient

It is easily seen from Figure 1, Appendix, that only $\cos \phi$ component of the lift is available as thrust. Therefore, the thrust due to lift of one blade at an azimuth angle of θ_z is

$$\overline{T}_{1_{\theta_z}} = \int_0^R \frac{\rho}{2} c_\ell \, c \, V^2 \cos \phi \, dr \tag{12}$$

The chord for a blade of any arbitrary shape may be represented by a power series in x. An expression of this type with the chord expressed as a percentage of the radius is

$$\frac{c}{R} = t_0 + t_1 x + t_2 x^2 + \cdot \cdot \cdot + t_n x^n \tag{13}$$

also since $x = r/R$ and $c_\ell = a\alpha$ the thrust of equation (12) can be written

$$\overline{T}_{1_{\theta_z}} = \frac{R^2 \, a \rho}{2} \int_0^1 (t_0 + t_1 x \quad . . .) \, V^2 \cos \phi \, dx \tag{14}$$

where $\alpha = \theta + \phi$ from the geometry.

This integral of equation (14) is usually integrated by assuming ϕ small so that $\cos\phi = 1$ and $\tan\phi = \phi$. However, this approximation is inaccurate for large values of ϕ. A better and more reasonable approximation is to assume, since α is small, that $\alpha = \sin\alpha$. This approximation would appear to be justified since, in practice, a rotor could not be operated with an appreciable amount of the important portions of the blades stalled. As $\alpha = \theta + \phi$ the approximation that $\alpha = \sin\alpha$ implies that $\alpha = \sin(\theta + \phi)$. Expanding $\sin(\theta + \phi)$ into

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi \qquad (15)$$

and replacing α in equation (14) by the expansion, the thrust for one blade at an azimuth angle of θ_z can be written

$$\bar{T}_{1_{\theta_z}} = \frac{R^2 a\rho}{2}\int_0^1 (t_o + t_1 x \; . \; . \; .)V^2 \cos^2\phi \sin\theta \, dx + \frac{R^2 a\rho}{2\,\cancel{\cos^2\theta \, dx}}\int_0^1 (t_o + t_1 x \; . \; . \; . \; .)V^2 \cos\phi \sin\phi \cos\theta \, dx \qquad (16)$$

however $V\sin\phi$ and $V\cos\phi$ are already known from equations (10) and (11).

Defining

$$C_{T_{1_{\theta_z}}} = \frac{\bar{T}_{1_{\theta_z}}}{\rho\pi\Omega^2 R^4} \qquad (17)$$

the equation may be written in the following form

$$\frac{2\pi C_{T_{I\theta_z}}}{a} = \int_0^1 (t_o + t_1 x \quad . \quad . \quad .)\ (v_x \sin \theta_z \quad x)^2 \sin \theta \, dx$$

$$+ \int_0^1 (t_o + t_1 x + . \ . \ .)\ (v_x \sin \theta_z + x) \left[v_z + v_x \cos \theta_z + x\ (w \cos \theta_z + y \sin \theta_z) \right] \cos \theta \, dx \tag{18}$$

For a linearly twisted blade the blade-section pitch angle is composed of four parts.

$$\theta = \theta_o + \theta_t x + \theta_{11} \sin \theta_z + \theta_{12} \cos \theta_z \tag{19}$$

where θ_o = extended blade root angle of incidence

θ_{11} = lateral component of cyclic pitch change

θ_{12} = longitudinal component of cyclic pitch change

θ_t = blade twist

Equation (18) may be now integrated with respect to x and the limits substituted. This gives the final thrust coefficient equation for one blade at an azimuth angle, θ_z.

where $$T_n = \frac{t_o}{n} + \frac{t_1}{n+1} + \frac{t_2}{n+2} + . \ . \ . \ . = \int_0^1 \frac{c}{R} x^{n-1} dx \tag{21}$$

for $n = 1, 2, 3, \ . \ . \ . \ n$

The thrust may be integrated over the whole range of azimuth angles (0 to 2π) to give the average thrust for a complete revolution. This may be seen to equal simply the constant parts of equation (20) with respect to θ_z.

$$\frac{2\pi}{a} G_{1\theta z} = \left[\frac{T_1}{2}(v_x - v_z\theta_{11})v_x - \frac{T_2}{2}\beta_0\theta_{12}v_x + \frac{T_3}{2}(2 - \theta_{11}y - \theta_{12}\omega) - \theta_t\left(v_z T_3 + \frac{y v_x T_3}{2}\right)\right]\sin\theta_0$$

$$+ \left[T_2\left[v_z + \left(\frac{y}{2} + \theta_{11}\right)v_x\right] + \theta_t\left(\frac{1}{2}v_x^2 T_2 + T_4\right)\right]\cos\theta_0$$

$$+ \sin\theta_z\left\{\left[-\frac{T_1}{4}\beta_0\theta_{12}v_x^2 + T_2\left(\left\{2 - \frac{3}{4}\theta_{11}y - \frac{\theta_{12}\omega}{4}\right\}v_x - \theta_{11}v_z\right) - \theta_t\left(v_z v_x T_2 + y T_4\right)\right]\sin\theta_0\right.$$

$$\left. + \left[T_1\left(\frac{3}{4}\theta_{11}v_x + v_z\right)v_x + T_3(y + \theta_{11}) + \theta_t 2 v_x T_3\right]\cos\theta_0\right\}$$

$$+ \cos\theta_z\left\{\left[-\frac{T_1}{4}\beta_0\theta_{11}v_x^2 - T_2\left(\left\{\frac{\theta_{11}\omega + \theta_{12}y}{4}\right\}v_x + \theta_{12}v_z\right) - \theta_t\left(v_x\beta_0 T_3 + \omega T_4\right)\right]\sin\theta_0\right.$$

$$\left. + \left[\frac{T_1}{4}\theta_{12}v_x^2 + T_2 v_x\beta_0 + T_3(\omega + \theta_{12})\right]\cos\theta_0\right\}$$

$$+ \sin 2\theta_z\left\{\left[-T_1(\theta_{11}\beta_0 + \theta_{12}v_z)v_x - T_3(\theta_{11}\omega + \theta_{12}y) - \theta_t\left(\frac{1}{2}v_x\beta_0^2 T_2 + \frac{1}{2}v_x\omega T_3\right)\right]\sin\theta_0\right.$$

$$\left. + \left[\frac{T_1}{2}v_x^2\beta_0 + T_2\left(\frac{\omega}{2} + \theta_{12}\right)v_x\right]\cos\theta_0\right\}$$

$$+ \cos 2\theta_z\left\{\left[\frac{T_1}{2}(\theta_{11}v_z - \theta_{12}\beta_0 - v_x)v_x + \frac{T_3}{2}(\theta_{11}y + \theta_{12}\omega) + \theta_t\left(\frac{v_x\beta_0^2 T_2}{2} + \frac{v_x\omega T_3}{2}\right)\right]\sin\theta_0\right.$$

$$\left. + \left[-T_2 v_x\left(\frac{y}{2} + \theta_{11}\right) - \theta_t\left(\frac{1}{2}v_x^2 T_2\right)\right]\cos\theta_0\right\}$$

$$+ \sin 3\theta_z\left\{\left[-\frac{T_1}{4}v_x\beta_0 + \frac{T_2}{4}(\theta_{11}y - \theta_{12}\omega)v_x\right]\sin\theta_0 + \left[-\frac{T_1}{4}\theta_{11}v_x^2\right]\cos\theta_0\right\}$$

$$+ \cos 3\theta_z\left\{\left[\frac{T_1}{4}\theta_{11}v_x^2\beta_0 + \frac{T_2}{4}(\theta_{11}\omega + \theta_{12}y)v_x\right]\sin\theta_0 + \left[-\frac{T_1}{4}\theta_{12}v_x^2\right]\cos\theta_0\right\}$$

Equation (20)

BLADE ROOT AIR MOMENT

Since the blade root air moment is equal to the negative of the same integral as the thrust with the exception that the power of x is one higher, all that is necessary to write the entire equation for the blade root air moment due to thrust is to change the sign and increase the subscripts of the T-factors by one in each term. The minus sign is introduced so that a positive blade root air moment produces a positive coning angle.

PITCHING MOMENT

The pitching moment is a moment summation about the y-axis. It is therefore a $\cos \theta_z$ component of the blade root air moment.

$$\frac{2\pi}{a} C_{My1_{\theta_z}} = \frac{2\pi}{a} C_{M1_{\theta_z}} \cos \theta_z \tag{22}$$

It may be easily obtained by multiplying the blade root air moment by $\cos \theta_z$ and expressing it in the desired form.

The blade root air moment is of the form:

$$\frac{2\pi}{a} C_{M1_{\theta_z}} = A' + B' \sin \theta_z + C' \cos \theta_z + D' \sin 2\theta_z + E' \cos 2\theta_z + F' \sin 3\theta_z + G' \cos 3\theta_z \tag{23}$$

Upon multiplying through by $\cos \theta_z$ it becomes the pitching moment.

$$\frac{2\pi}{a} C_{M_{y1}\theta_z} = A' \cos\theta_z + B' \cos\theta_z \sin\theta_z + C' \cos^2\theta_z + D' \cos\theta_z \sin 2\theta_z + E' \cos\theta_z \cos 2\theta_z + F' \cos\theta_z \sin 3\theta_z + G' \cos\theta_z \cos 3\theta_z \qquad (24)$$

Then if again expressed in terms of multiple angles

$$\frac{2\pi}{a} C_{M_{y1}\theta_z} = \frac{1}{2}\left[C' + D' \sin\theta_z + (2A' + E') \cos\theta_z + (B' + F') \sin 2\theta_z + (C' + G') \cos 2\theta_z + D' \sin 3\theta_z + E' \cos 3\theta_z + F' \sin 4\theta_z + G' \cos 4\theta_z \right] \qquad (25)$$

where A', B', C', D', E', F', and G' are the coefficients of the respective harmonics as given in the blade root air moment equation.

ROLLING MOMENT

The rolling moment is found from the $-\sin\theta_z$ component of the blade root air moment. Therefore as before

$$-\frac{2\pi}{a} C_{M_{x1}\theta_z} = \frac{2\pi}{a} C_{M_1} \sin\theta_z \qquad (26)$$

Multiplying by $\sin\theta_z$ gives

$$-\frac{2\pi}{a} C_{M_{x1}\theta_z} = A' \sin\theta_z + B' \sin^2\theta_z + C' \sin\theta_z \cos\theta_z + D' \sin\theta_z \sin 2\theta_z + E' \sin\theta_z \cos 2\theta_z + F' \sin\theta_z \sin 3\theta_z + G' \sin\theta_z \cos 3\theta_z \qquad (27)$$

Then in terms of the multiple angles

$$-\frac{2\pi}{a}C_{M_{x_1\theta_z}} = \frac{1}{2}\Big[B' + (2A' - E')\sin\theta_z + D'\cos\theta_z + (C' - G')\sin 2\theta_z + (F' - B')\cos 2\theta_z + E'\sin 3\theta_z + G'\sin 4\theta_z - F'\cos 4\theta_z - D'\cos 3\theta_z\Big] \qquad (28)$$

Inspection of the pitching moment and rolling moment equations shows that they are composed of a constant part and four harmonics of the azimuth angle. The blade root air moment equation has a constant part and three harmonics. The average value of these moments over one complete revolution is simply the constant parts of the equation. It is these average values that are required to be in equilibrium with the the external forces and moments.

SOLUTION OF THE EQUATIONS

The constant parts of the thrust, blade root moment, rolling moment, and pitching moment equations are to be solved for θ_o, θ_{11}, θ_{12}, and β_o. For steady equilibrium flight the pitching and rolling moment coefficients must be zero. Since the equations are transcendental and non-linear they do not have a simple solution. For this reason a method of sucessive approximations is used to solve the equations. β_o can be approximated to a close degree by assuming the thrust center to be at the three-quarter radius of the blade. A plot of β_o vs. C_T is shown in Figure (2) for this approximation. Using this approximate value of β_o various values of θ_o can then be selected, and the equilibrium equations can be solved for C_{T1} and C_{M1}. A plot of θ_o vs C_T will enable the correct θ_o to be chosen for the known value of C_{T_1}. To see if the assumed value of β_o is sufficiently close for accuracy requires a moment summation about the flapping hinge.

$$\Sigma M_\beta = 0 = M_{CF} + M_W + M_1 \tag{29}$$

where

M_{CF} 2B. is the moment due to centrifugal force $-\frac{1}{2} I_\beta \Omega^2 \sin 2\beta_o$

M_W is the moment due to the blade dead weight

M_1 is the blade root air moment $C_{M_1} \rho \pi \Omega^2 R^5$

The magnitude of M_W is usually small in comparison to M_{CF} and M_1 and can be neglected in many cases.

If the β_o as found by this moment summation is seen to be reasonably close to the originally assumed value of β_o, θ_{11} and θ_{12} can be found using the assumed β_o. The new β_o can be used, if necessary, to obtain greater accuracy in the solution.

The reason this method can be employed in solving the equilibrium equations is that the coning angle, β_o, is the least important of the unknowns since a small change in it does not effect θ_{11}, θ_{12}, and θ_o appreciably.

The above procedure is outlined as follows:

(1) Pick a β_o from Figure (2), appendix.

(2) Assume values of θ_o and solve for C_{T_1} and C_{M_1}

θ_o may be approximated by

$$\theta_o = \frac{\frac{2\pi}{a} C_{T_1} - \theta_t \left(T_4 + 2C_2\right) + \left(2E_2 - V_2 T_2\right)}{2C_1 + T_3} \qquad (30)$$

(3) Plot C_T vs θ_o and from this plot find θ_o corresponding to the desired C_{T_1}.

(4) Find β_o by a moment summation about the flapping axis. This value of β_o should compare favorably with the assumed value. If not, this new β_o should be used in the next solution.

(5) Solve for θ_{11} and θ_{12}. The values of θ_{11}, θ_{12}, and β_o desired are at the value of θ_o which satisfies the thrust requirement.

The equations may be simplified if the general terms are collected and represented by one coefficient.

Terms that occur frequently are

$$A_n = -\frac{1}{4}\left(T_n \beta_0 v_x - T_{n+1} w\right) \tag{31}$$

$$B_n = v_x T_n \tag{32}$$

$$C_n = \frac{1}{2} v_x^2 T_n \tag{33}$$

$$D_n = -\frac{1}{2}\left(v_x v_z T_n + y\, T_{n+2}\right) \tag{34}$$

$$E_n = -\frac{1}{4} y\, T_n v_x \tag{35}$$

where n = 1, 2, 3, . . .n

If further simplification is made by using

$$L = (3E_3 - v_z T_3)\theta_{os} + (3C_2 + T_4)\theta_{oc} \tag{36}$$

$$M = (E_3 - v_z T_3)\theta_{os} + (C_2 + T_4)\theta_{oc} \tag{37}$$

$$N = D_1 \theta_{os} + B_2 \theta_{oc} \tag{38}$$

$$P = D_2 \theta_{os} + B_3 \theta_{oc} \tag{39}$$

$$K_R = -(2B_3 + \theta_t 2D_3)\theta_{os} - (-2D_2 + \theta_t 2B_4)\theta_{oc} \tag{40}$$

$$K_P = -\theta_t 4A_4 \theta_{os} + 4A_3 \theta_{oc} \tag{41}$$

$$K_T = -\left[2C_1 + T_3 + \theta_t(2E_3 - v_z T_3)\right]\theta_{os} - \left[-(2E_2 - v_z T_2) + \theta_t(2C_2 + T_4)\right]\theta_{oc} \tag{42}$$

$$K_M = -\left[2C_2 + T_4 + \theta_t(2E_4 - v_z T_4)\right]\theta_{os} - \left[-(2E_3 - v_z T_3) + \theta_t(2C_3 + T_5)\right]\theta_{oc} \tag{43}$$

where $\theta_{os} = \sin\theta_o$ and $\theta_{oc} = \cos\theta_o$

Introducing these coefficients the four equations may be written as

<u>Rolling Moment Equation</u>

$$A_2 v_x \theta_{os} \theta_{12} + L\theta_{11} = K_R \tag{44}$$

<u>Pitching Moment Equation</u>

$$M\,\theta_{12} \qquad + A_2\, v_x\, \theta_{os}\, \theta_{11} \qquad = K_P \tag{45}$$

<u>Thrust Equation</u>

$$2A_2\, \theta_{os}\, \theta_{12} + N\, \theta_{11} - \frac{2\pi}{a} C_{T_1} \qquad = K_T \tag{46}$$

<u>Blade Root Air Moment Equation</u>

$$2A_3\, \theta_{os}\, \theta_{12} + P\, \theta_{11} \qquad + \frac{2\pi}{a} C_{M_1} = K_M \tag{47}$$

which for an assumed and θ_o gives for the general solution

$$C_{T_1} = \frac{-\dfrac{a}{2\pi}\begin{vmatrix} A_2 v_x \theta_{os} & L & K_R \\ M & A_2 v_x \theta_{os} & K_P \\ 2A_2\theta_{os} & N & K_T \end{vmatrix}}{\begin{vmatrix} A_2 v_x \theta_{os} & L \\ M & A_2 v_x \theta_{os} \end{vmatrix}} \tag{48}$$

$$C_{M_1} = \frac{\dfrac{a}{2\pi}\begin{vmatrix} A_2 v_x \theta_{os} & L & K_R \\ M & A_2 v_x \theta_{os} & K_P \\ 2A_2\theta_{os} & N & K_M \end{vmatrix}}{\begin{vmatrix} A_2 v_x \theta_{os} & L \\ M & A_2 v_x \theta_{os} \end{vmatrix}} \tag{49}$$

$$\theta_{12} = \frac{\begin{vmatrix} K_R & L \\ K_P & A_2 v_x \theta_{os} \end{vmatrix}}{\begin{vmatrix} A_2 v_x \theta_{os} & L \\ M & A_2 v_x \theta_{os} \end{vmatrix}} \tag{50}$$

$$\theta_{11} = \frac{\begin{vmatrix} A_2 v_x \theta_{os} & K_R \\ M & K_P \end{vmatrix}}{\begin{vmatrix} A_2 v_x \theta_{os} & L \\ M & A_2 v_x \theta_{os} \end{vmatrix}} \tag{51}$$

CONCLUSIONS

(1) The equilibrium angles obtained by the solution of the equations developed in this paper appear to give better agreement with experimental helicopter data than previous equations based on the approximation that ϕ is a small angle.

(2) The approximation that α is a small angle which was used to simplify the equations is more dependable than the approximation that the inflow angle remain small.

(3) The T-factors introduced in this paper to describe the rotor blade planform are exact. Thus the usual error introduced by the previous use of an effective solidity is eliminated.

(4) The theory should not only be applicable to helicopter rotors at any rotor angle of attack, but also to convertaplane rotors and propellers in yaw where the advance ratio may be quite big.

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APPENDIX

SAMPLE PROBLEM

The data for this sample problem is taken from TM 1266. Run #13 gives the following data:

V_h	62 feet per second
Ω	22.4 radians per second
v_x	.143
C_T	.00588
α_K	-10.4°

The following parameters are calculated

$\overline{V}_i$	-8.35 feet per second
v_z	-.046
y	.0050
w	-.0143
β_o	-9.5° from Figure (2), Appendix
θ_o	9.65° by equation (30)

Using the above data Tables I, II, and III were completed. Table IV shows the results of this theory with experiment and also with the calculated results of the theory used with TM 1266.

TABLE I Calculated Coefficients

n	T_n	A_n	B_n	C_n	D_n	E_n
1	.0610	.000480	.00875	.000314	.000150	-.00001095
2	.0325	.000267	.00466	.000167	.000071	-.00000583
3	.0204	.000173	.00293	.000105		-.00000366
4	.0144	.000125	.00207			
5	.0110					
6	.0087					

TABLE II Calculated Coefficients

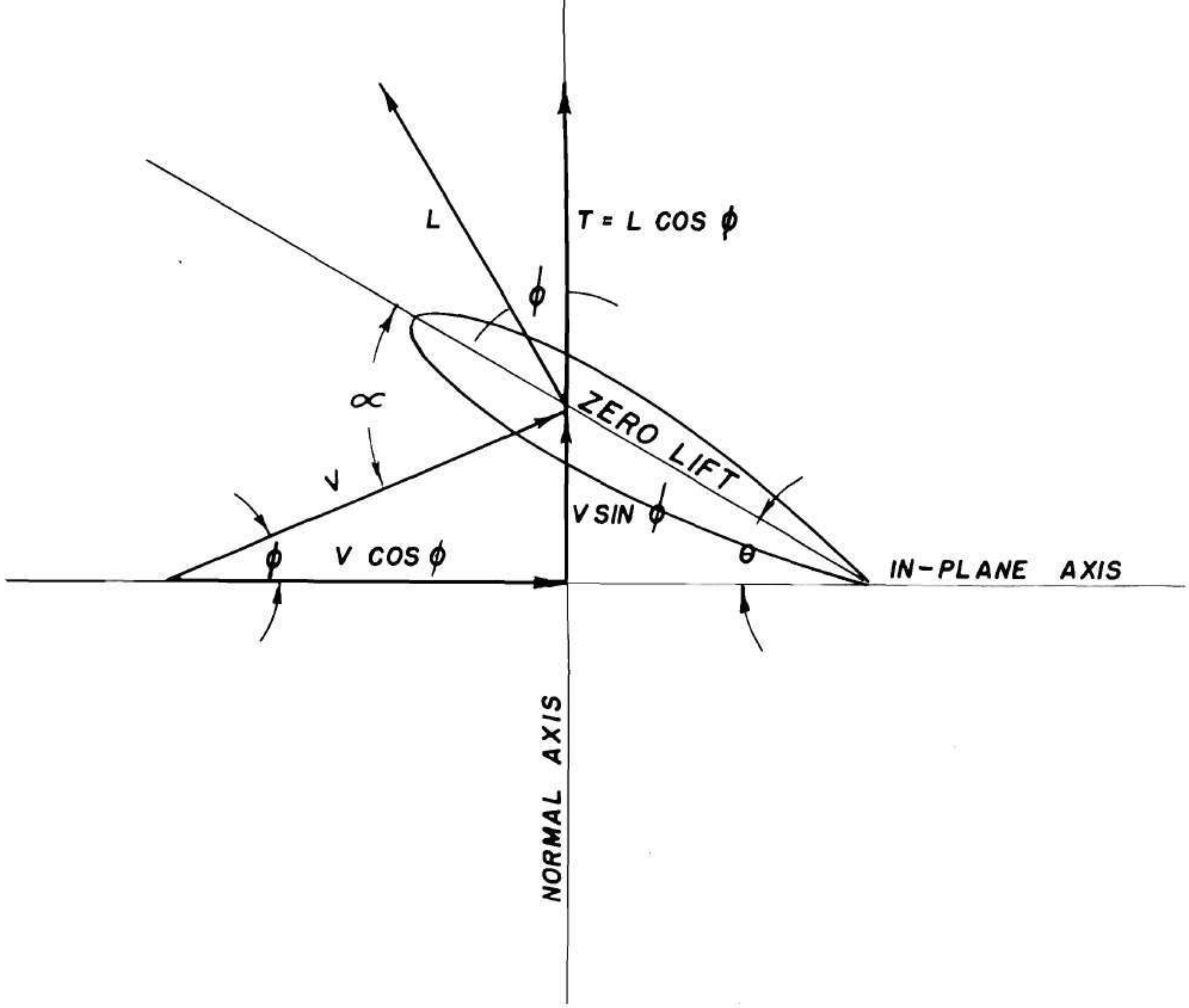
θ_o	L	M	N	P	K_R	K_P	K_T	K_M
9	.014862	.014533	.004626	.002905	-.000777	.000683	-.oo1824	-.001386
10	.014836	.014507	.004615	.002897	-.000877	.000681	-.002190	-.001641
11	.014804	.014477	.004603	.002889	-.000979	.00679	-.002556	-.001897

TABLE III Results for Various θ_o

θ_o	C_{T_3}	B_o	θ_{11}	θ_{12}
9	.00456	7.91	3.00	2.69
10	.00552	9.50	3.39	2.69
11	.00650	11.10	3.79	2.69

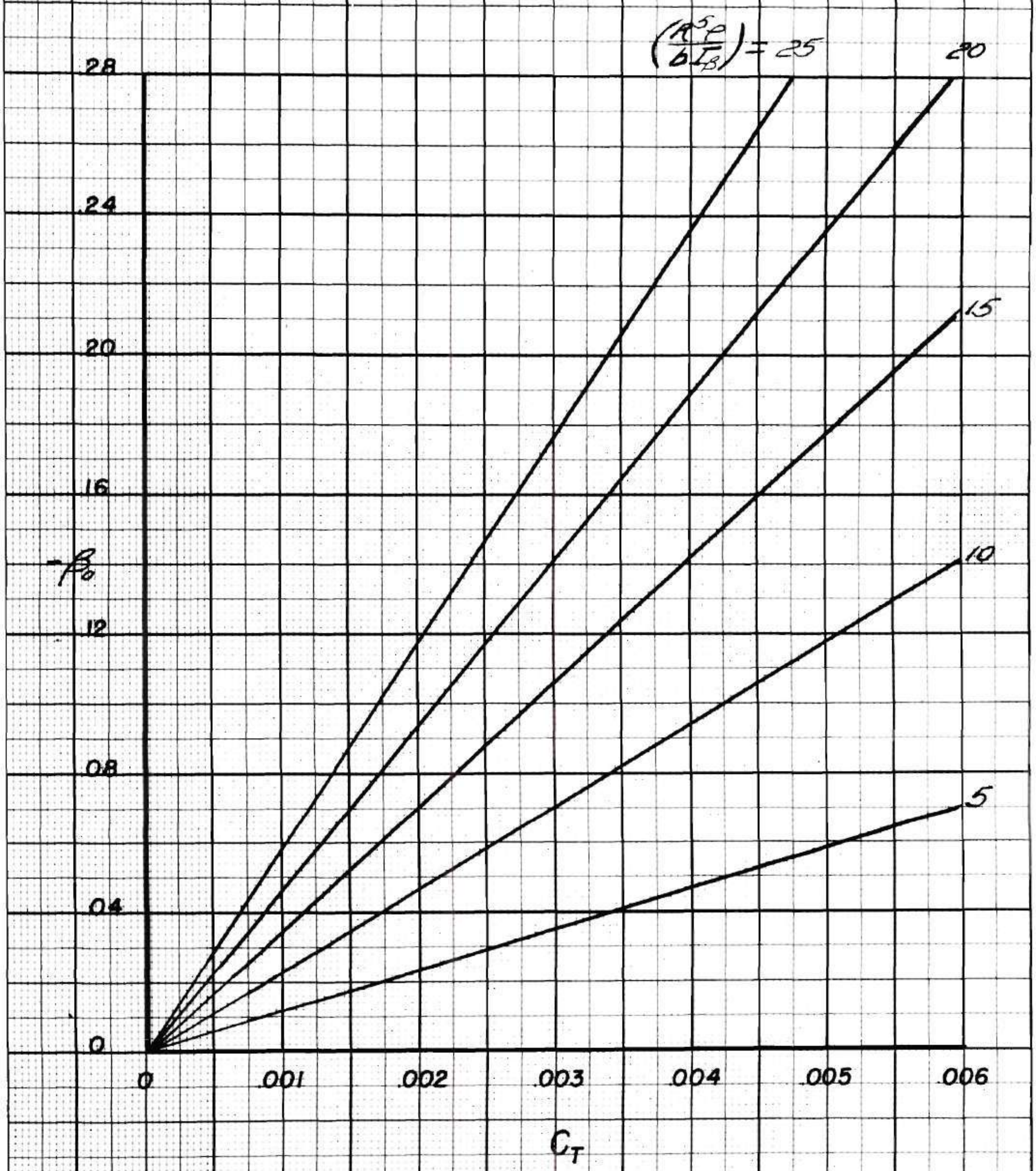
TABLE IV Comparison of Theory

	Theory	Experiment	TM 1266 Theory
θ_o	10.35	10.40	11.4
θ_{11}	-3.52	-4.40	-3.42
θ_{12}	2.69	3.50	1.76
B_o	-10.1	-9.63	-8.96



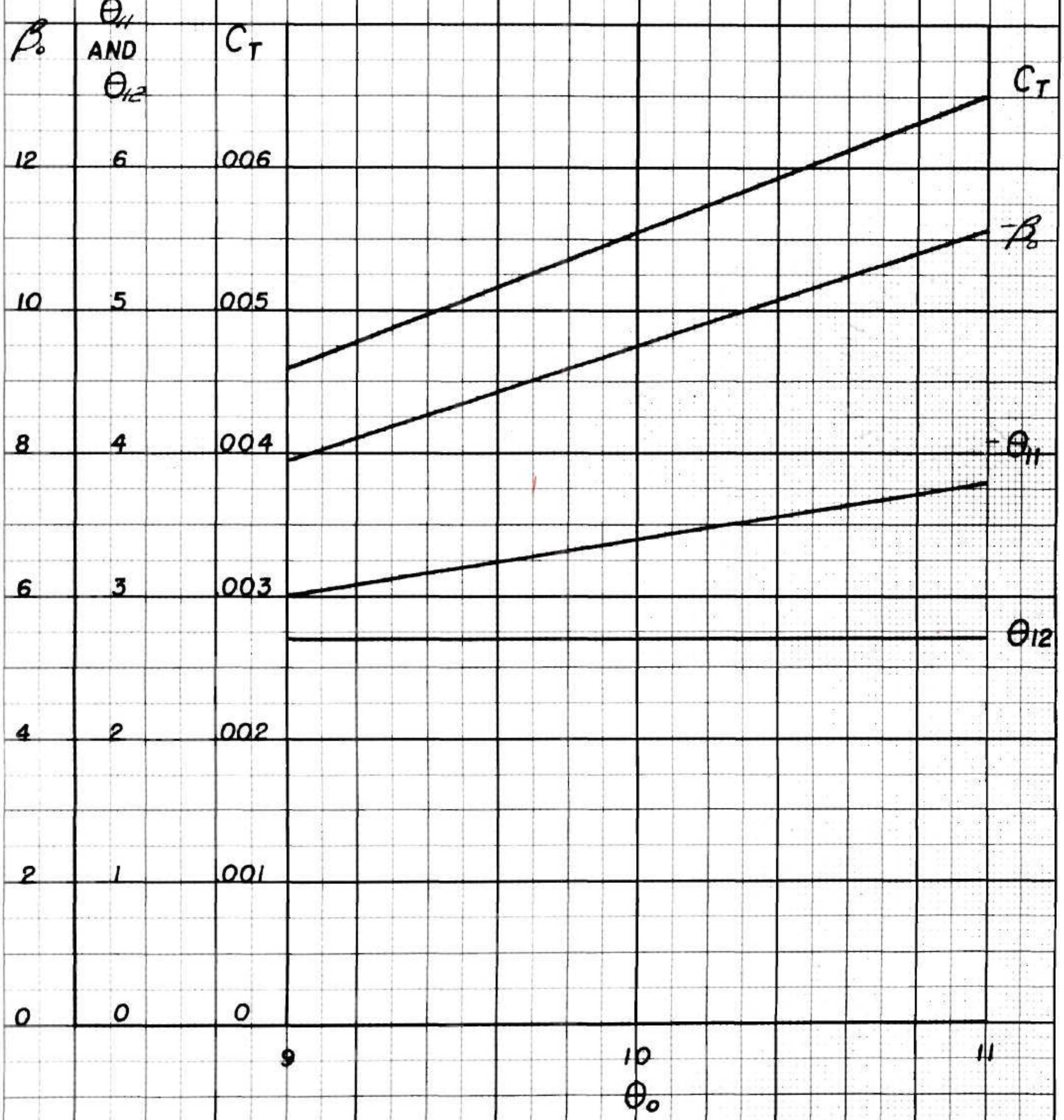
VELOCITY COMPONENTS ACTING ON A BLADE ELEMENT

Figure 1



APPROXIMATION FOR CONING ANGLE

Figure 2



PLOT OF θ_{11}, θ_{12}, β_0 AND C_T VS θ_0 FOR SAMPLE PROBLEM

Figure 3